Maps, Bridges, Networks, and Art Galleries: Introducing Secondary Students to Graph Theory through Classic Problems

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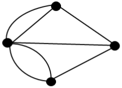
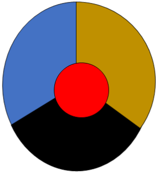
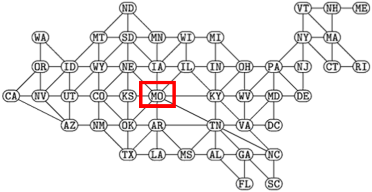
Context of Courses

- 50 minute classes
- Middle or high school students
- Bridges, Maps, and Networks: broad introduction to graph theory
- How to (Mathematically) Guard an Art Gallery: introducing and using graph theory with one problem as motivation
- Prerequisites: Comfort with using variables, reading algebraic expressions

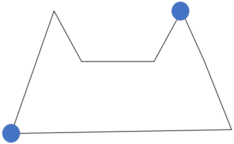
Why Classic Problems?







Eric W. Weisstein, Contiguous USA Graph



- Accessible
- Mix familiar and new ideas
- Helped establish or build subfields

Bridges, Maps, Networks Class Structure

Bridges of Königsberg problem. Definitions of graph, vertex, edge, degree, cycle, Eulerian path/cycle. → Routing problems. Structures in graphs.

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4 Color Theorem, map that needs 4 colors. Definition of planar graphs. 5 Color Theorem partial proof. → Chromatic graph theory.

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What graphs could represent. Different measures of centrality. Random graphs as models. Spread. → Network theory.

Bridges, Maps, Networks Student Survey Responses

Favorite Parts:

- **Bridges (5/29):** "bridge/island examples," "applying a real-world situation (bridges) first before learning about graph theory"
- **5-Color Theorem Proof (8/29):** "the revelation at flipping the colors of 2 and 4"
- **Other Aspects of Maps (4/29):** "drawing maps and then seeing the connection between those maps and graphs," "creating an example that needed four colors"
- **Networks (6/29):** "learning measures of centrality,"
- **Hands-on aspect (5/29):** "figuring out certain problems then discussing their solutions"

Bridges, Maps, Networks Student Survey Responses

Recommendations for Changes:

- **Pace (12/29):** "wish we had more time to talk about networks," "the proof for 5 colors was explained too quickly"
- **Handout (3/29):** "have resource list," "handout so it's easier to follow"
- **Interaction (3/29):** "have more puzzles, especially for networks part," "maybe more student interaction?"

How to (Mathematically) Guard an Art Gallery

How many 360° cameras placed at the gallery's vertices are necessary to guard an n-gonal art gallery?

Examples and key proof idea.

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Triangulation, trees, and duals.

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Color vertices and place cameras.

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Discuss variations of problem.

Art Gallery Student Survey Responses

Favorite Parts:

- **Particular Mathematical Ideas (12/26, 6/15):** "the discussion of triangulation and how [the dual] must always be a tree"
- **Finale (5/26, 2/15):** "when I realized that you can just put a camera at each of one color," "how it comes together in the end"

Recommendations for Changes:

- **Pace (2/26, 3/15):** "the first part of the class could have gone faster and the second gone slower"
- **More depth or examples (5/26, 6/15):** "Would be interesting to try for a set of definite shapes," "more depth for the additional questions"