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L(d,1)-labelings of the edge-path-replacement by factorization of graphs

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Abstract For an integer $d \geq 2$, an $L(d,1)$-*labeling* of a graph G is a function f from its vertex set to the non-negative integers such that $|f(x) - f(y)| \geq d$ if vertices x and y are adjacent, and $|f(x) - f(y)| \geq 1$ if x and y are at distance two. The minimum span over all the L(d,1)-labelings of G is denoted by $\lambda_d(G)$. For a given integer $k \geq 2$, the *edge-path-replacement of* G or $G(P_k)$ is the graph obtained from G by replacing each edge with a path P_k on k vertices. We show that the edges of G can be colored with $\lceil \Delta(G)/2 \rceil$ colors so that each monochromatic subgraph has maximum degree at most 2 and use this fact to establish general upper bounds on $\lambda_d(G(P_k))$ for $k \geq 4$. As a corollary, we settle the following conjecture by Lü (J Comb Optim, 2012): for any graph G with $\Delta(G) \geq 2$, $\lambda_2(G(P_4)) \leq \Delta(G) + 2$. Moreover, $\lambda_2(G(P_4)) = \Delta(G) + 1$ when $\Delta(G)$ is even and different from 2. We also show that the class of graphs $G(P_k)$ with $k \geq 4$ satisfies a conjecture by Havet and Yu (2008 Discrete Math 308:498–513) in the related area of $(d, 1)$-total labeling of graphs.

Keywords L(2,1)-labeling · L(d,1)-labeling · $(d,1)$-Total labeling · Edge-path-replacement · Factorization of graphs

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1 Introduction

When allocating frequency bands among spatially distributed transmitters, one may reduce interference by assigning bands which are sufficiently spectrally separated to transmitters in close proximity to one another. In one graph theoretical model of this problem, each vertex represents a transmitter, each edge connects vertices associated to transmitters that are sufficiently close, and the bands of the frequency spectrum are represented by a collection of non-negative integers. To reduce interference when allocating frequency bands, we assign a non-negative integer to each vertex such that any two adjacent vertices receive integers at least $d \geq 2$ apart, and vertices at distance two receive integers at least one apart. More formally, an *$L(d,1)$-labeling* of G is a function f from the vertex set to the non-negative integers such that $|f(x)-f(y)| \geq d$ if vertices x and y are adjacent (*distance one condition*), and $|f(x) - f(y)| \geq 1$ if x and y are at distance two (*distance two condition*). L(d,1)-labelings have been studied extensively since the introduction of L(2,1)-labelings in Griggs and Yeh (1992) and continue to generate a rich literature as corroborated by several articles recently published or to appear (Adams et al. 2012; Cerioli and Posner 2012; Charpentier et al. 2012; Chia et al. 2012; Havet et al. 2012; Lin and Wu 2012; Lü 2012; Lü and Lin 2012; Panda and Goel 2012; Wang and Lin 2012; Wu et al. 2012; Zhai et al. 2012). For an overview on the subject, we refer the reader to the surveys in (Calamoneri 2011) and (Yeh 2006).

The minimum span over all the L(d,1)-labelings of a graph G will be denoted by $\lambda_d(G)$. Griggs and Yeh (1992) conjectured that $\lambda_2(G) \leq \Delta^2(G)$ where $\Delta(G)$ denotes the maximum degree of G. This conjecture holds for $\Delta(G) \geq 10^{69}$ (Havet et al. 2012; Calamoneri 2011) but the best general upper bound yet established is $\lambda_2(G) \leq \Delta^2(G)+\Delta(G)-2$ (Gonçalves 2008). As the general problem of determining $\lambda_2(G)$ is NP-hard (Georges et al. 1994), it is of interest to find bounds or exact values for $\lambda_d(G)$ within certain classes of graphs. For instance, the L(2,1)-labelings of the edge-path-replacement of graphs were first investigated by Lü (2012) and further generalized to L(d,1)-labelings by Lü and Lin (2012). For $k \geq 2$, *the edge-path-replacement* $G(P_k)$ of a graph G is a graph obtained by replacing each edge with a path P_k on k vertices. In (Lü 2012) and (Lü and Lin 2012), bounds and some exact values for $\lambda_d(G(P_k))$ were obtained for different families of graphs and several values of d and k. In particular, the bound $\lambda_2(G(P_4)) \leq \Delta(G) + 4$ was established in (Lü 2012) and later improved to $\lambda_2(G(P_4)) \leq \Delta(G) + 3$ by the more general result $\lambda_d(G(P_4)) \leq \Delta(G) + 2d - 1$ if $\Delta(G) \geq 3$ in (Lü and Lin 2012). However, these bounds failed to be tight for a significant number of graphs even when $d = 2$. For example, if G is a tree, wheel, Möbius ladder, or the Cartesian products of two paths, of two cycles, of a cycle and a path, or of two complete graphs, then $\lambda_2(G(P_4))$ is either $\Delta(G)+1$ or $\Delta(G)+2$ (Lü 2012). The same paper also proves that, for all $k \geq 5$, the bound $\lambda_2(G(P_k)) \leq \Delta(G) + 2$ holds for any graph with $\Delta(G) \geq 2$, hence the following natural conjecture was proposed:

Conjecture 1.1 (Lü 2012) For any graph G with $\Delta(G) \geq 2$, $\lambda_2(G(P_4)) \leq \Delta(G)+2$.

Closely related to the L(d,1)-labeling is the (d,1)-total labeling studied in (Havet and Yu 2008). A *(d,1)-total labeling* of G is a function g from the union of the vertex and edge sets to the non-negative integers such that $|g(x) - g(y)| \geq d$ if vertex x is incident to edge y, and $|g(w) - g(z)| \geq 1$ if w and z are two adjacent vertices or two incident edges. The minimum span over all the (d,1)-total labelings of a graph G is denoted by $\lambda_d^T(G)$ and is linked to the minimum span over all the L(d,1)-labelings through the equalities $\lambda_d^T(G(P_k)) = \lambda_d(G(P_{2k-1}))$ for $k \geq 2$ (Lü 2012). Note that $\lambda_d^T(G) = \lambda_d^T(G(P_2)) = \lambda_d(G(P_3))$. The following conjecture was proposed:

Conjecture 1.2 (Havet and Yu 2008) For any graph G, $\lambda_d^T(G) \leq \Delta(G) + 2d - 1$.

In Sect. 2, we use a classic result in the area of graph factorizations to show that any graph G admits an edge-coloring using $\lceil \Delta(G)/2 \rceil$ colors so that each monochromatic subgraph has maximum degree at most 2. We then use this fact to establish general upper bounds on $\lambda_d(G(P_k))$ for $k \geq 4$. These bounds are used to verify that Conjecture 1.1 is true. In addition, if $\Delta(G)$ is even and different from 2, then $\lambda_2(G(P_4)) = \Delta(G) + 1$. The same bounds are also used to show that the graphs $G(P_k)$ with $k \geq 4$ for any graph G satisfy Conjecture 1.2.

2 Upper bounds for $\lambda_d(G(P_k))$ for $k \geq 4$

In this section we will show that Conjecture 1.1 is true and that the graphs $G(P_k)$ with $k \geq 4$ for any graph G satisfy Conjecture 1.2. We first provide some preliminary definitions and a result on factorization of graphs.

A subgraph F of a graph G is a *factor of* G if F is spanning in G. If the edge set of G can be represented as an edge-disjoint union of factors $F_1, F_2, \ldots, F_h$, we shall refer to this set of factors as a *factorization* of G; in addition, if every vertex in each F_i, for $i = 1, 2, \ldots, h$, has degree r, we call this factorization an *r-factorization of* G. The following result provides a complete characterization of graphs with a 2-factorization:

Result 2.1 (Petersen 1891) A graph G has a 2-factorization if and only if G is a regular graph and $\Delta(G)$ is even.

We present an edge coloring of graphs in Lemmas 2.2 and 2.3 that will be useful in Theorem 2.4.

Lemma 2.2 *Let G be an arbitrary graph. Then there exists a regular graph G' so that G is a subgraph of G' with $\Delta(G') = \Delta(G)$ if $\Delta(G)$ is even, and $\Delta(G') = \Delta(G) + 1$ if $\Delta(G)$ is odd.*

Proof Let $p = \Delta(G)$ if $\Delta(G)$ is even, and $p = \Delta(G) + 1$ if $\Delta(G)$ is odd. Since there is an even number of odd degree vertices in G, for each pair of vertices u, w of odd degrees, we add a new copy of $K_{p+1} - e$ (the complete graph on $p + 1$ vertices without a single edge e), connect u to one of the two vertices of degree $p - 1$ in this copy, and connect w to the other vertex of degree $p - 1$. In the new graph, all vertices have even degrees and the maximum degree is p. For each vertex v with degree deg(v) in the new graph, add $q = (p - \deg(v))/2$ pairwise disjoint copies of $K_{p+1} - e$ and connect v to the two vertices of degree $p - 1$ in each of these q copies. The resulting graph G' is regular, contains G as a subgraph, and has $\Delta(G') = p$. □

Lemma 2.3 *Let G be an arbitrary graph. Then it is possible to color the edges of G with $\lceil \Delta(G)/2 \rceil$ colors so that each monochromatic subgraph has maximum degree at most* 2.

Proof Use Lemma 2.2 to construct a regular graph G' so that G is a subgraph of G' with $\Delta(G') = \Delta(G)$ if $\Delta(G)$ is even, and $\Delta(G') = \Delta(G) + 1$ if $\Delta(G)$ is odd. By Result 2.1, G has a 2-factorization $F_1, F_2, \ldots, F_h$. Since each F_i is a spanning subgraph of G with vertices of degree 2 for $i = 1, 2, \ldots, h$, and since every vertex has even degree $\Delta(G')$, we conclude that $h = \Delta(G')/2 = \lceil \Delta(G)/2 \rceil$. Color each edge e of G with color i if e belongs to F_i. This edge-coloring uses at most h colors. To show that exactly h colors were used, observe that a vertex v of degree $\Delta(G)$ in G is also a vertex of degree $\Delta(G')$ in G'; since $\Delta(G)$ and $\Delta(G')$ differ by at most one, the edges of G incident to v will use at least h colors. □

Theorem 2.4 *Let G be an arbitrary graph and let $d \geq 2$ be an integer. Then*

i. $\lambda_d(G(P_4)) \leq d + 2$ *if $\Delta(G) \leq 2$, and*
ii. $\lambda_d(G(P_4)) \leq d + \lceil \Delta(G)/2 \rceil + \max\{\lceil \Delta(G)/2 \rceil, d\} - 1$ *otherwise.*

Proof First note that $\Delta(G) = \Delta(G(P_4))$. If $\Delta(G) \leq 2$, then $G(P_4)$ is a disjoint union of paths and cycles and hence the required upper bound holds since it is known that $\lambda_d(H) \leq d + 2$ if H is a path or a cycle (Griggs and Yeh 1992; Georges and Mauro 1995).

Suppose on the other hand that $\Delta(G) \geq 3$ and set $h = \lceil \Delta(G)/2 \rceil$. In view of Lemma 2.3, color the edges of G with h colors so that each monochromatic subgraph has maximum degree at most 2, and let $G_1, G_2, \ldots, G_h$ be these subgraphs (they are not necessarily spanning). Note that for each $i = 1, 2, \ldots, h$ there is a natural isomorphism between $G_i(P_4)$ and its corresponding subgraph of $G(P_4)$, so we will also call this subgraph $G_i(P_4)$ for the sake of simplicity; for similar reasons, we will also say that a vertex in $G(P_4)$ is in G if it corresponds to an original vertex in G.

In $G(P_4)$, label all the vertices in G with 0. The remaining vertices in $G(P_4)$ will be labeled as follows. For a fixed $i = 1, 2, \ldots, h$, each nontrivial path and each cycle in $G_i(P_4)$ is a sequence of vertices that repeats the following ordered pattern: a vertex in G followed by two vertices that are not in G and not in any $G_j(P_4)$ for $j \neq i$. Letting $m = \max\{\lceil \Delta(G)/2 \rceil, d\}$, each occurrence of this pattern is labeled with the three labels 0, $(d + i - 1)$, $(d + m + i - 1)$, respectively, in order, from left to right along each path and clockwise around each cycle. Note that each vertex in $G_i(P_4)$ that is also in G maintains its original label 0. All the vertices in $G(P_4)$ get assigned a label as summarized in Table 1. (In Fig. 1, we provide an example of this construction.)

Let H be either a path or a cycle in $G_i(P_4)$ for some $i = 1, 2, \ldots, h$. By inspection of the second column of Table 1, the labels used in H induce an L(d,1)-labeling of H since both distance conditions are satisfied within H (recall $m = \max\{\lceil \Delta(G)/2 \rceil, d\} \geq d$). Since each monochromatic subgraph of G has maximum degree at most 2, the vertices labeled 0 in the original graph G can only belong to at most one connected component of each $G_i(P_4)$ for $i = 1, 2, \ldots, h$. All the labels in the second column of Table 1 that will be assigned to vertices adjacent to the vertices labeled 0 in $G(P_4)$ are different, so these labels do not violate the distance two condition. Therefore, we

Table 1 Labeling summary in the proof of Theorem 2.4 for $G(P_4)$ where $h = \lceil \Delta(G)/2 \rceil$ and $m = \max\{\lceil \Delta(G)/2 \rceil, d\}$

Subgraph of $G(P_4)$	Repeating label pattern for paths/cycles		
$G_1(P_4)$	0,	d,	$d+m$
$G_2(P_4)$	0,	$d+1$,	$d+m+1$
$G_3(P_4)$	0,	$d+2$,	$d+m+2$
…	…	…	…
$G_{h-1}(P_4)$	0,	$d+h-2$,	$d+h+m-2$
$G_h(P_4)$	0,	$d+h-1$,	$d+h+m-1$

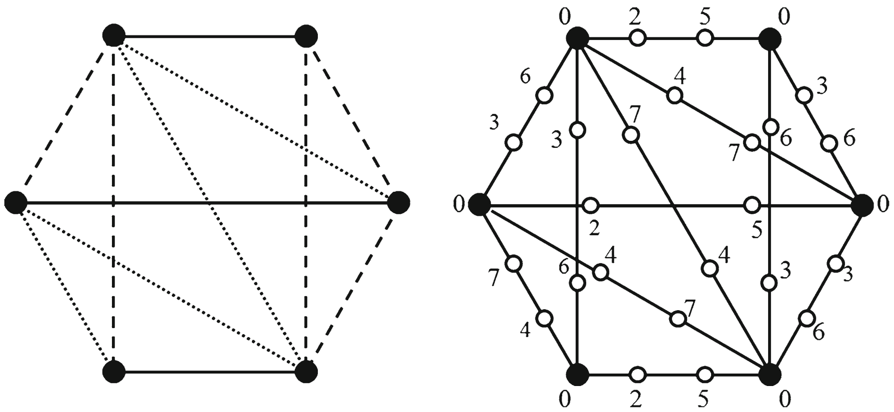

Fig. 1 The graph G on the *left* shows an edge-coloring satisfying Lemma 2.3 (*colors* represented by *solid*, *dashed*, and *dotted edges*, respectively); the graph on the *right* shows the L(2,1)-labeling of $G(P_4)$ constructed in the proof of Theorem 2.4.

have constructed an L(d,1)-labeling of $G(P_4)$ using labels in 0, 1, …, $d+h+m-1$, and so $\lambda_d(G(P_4)) \leq d+h+m-1$. □

Corollary 2.5 *Let G be an arbitrary graph and let d* ≥ 2 *be an integer. For any integer* $q \geq 0$,

i. $\lambda_d(G(P_{3q+4})) \leq d+2$ *if* $\Delta(G) \leq 2$, *and*
ii. $\lambda_d(G(P_{3q+4})) \leq d + \lceil \Delta(G)/2 \rceil + \max\{\lceil \Delta(G)/2 \rceil, d\} - 1$ *otherwise.*

Proof Let q be a nonnegative integer. If $\Delta(G) \leq 2$, then $\lambda_d(G(P_{3q+4})) \leq d+2$ by an argument similar to the one used in the proof of Theorem 2.4 i. Suppose on the other hand that $\Delta(G) \geq 3$ and consider the labeling of $G(P_4)$ constructed in the proof of Theorem 2.4 ii. For a fixed edge e of G, let $(0, x, y)$ be the label pattern in the second column of Table 1 which was used to label the first 3 vertices in the P_4 that replaced e in $G(P_4)$; repeat this same label pattern $q+1$ times to label the first $3q+3$ vertices of P_{3q+4} that replaces e in $G(P_{3q+4})$, in order, from left to right along the path, and label the final vertex in this path with a 0. As in the proof of Theorem 2.4 ii., it can be shown that this labeling of $G(P_{3q+4})$ is an L(d,1)-labeling and $\lambda_d(G(P_{3q+4})) \leq d + \lceil \Delta(G)/2 \rceil + \max\{\lceil \Delta(G)/2 \rceil, d\} - 1$. □

Corollary 2.6 *Let G be a graph with* $\lceil \Delta(G)/2 \rceil \geq d \geq 2$. *For any integer* $q \geq 0$, $\lambda_d(G(P_{3q+4})) \leq \Delta(G) + d$; *if in addition* $\Delta(G)$ *is even, then* $\lambda_d(G(P_{3q+4})) = \Delta(G) + d - 1$.

Proof Let q be a nonnegative integer. Note that $\Delta(G) \geq 3$ since $\lceil \Delta(G)/2 \rceil \geq d \geq 2$. From Corollary 2.5 we have that $\lambda_d(G(P_{3q+4})) \leq d + 2\lceil \Delta(G)/2 \rceil - 1$. Therefore $\lambda_d(G(P_{3q+4})) \leq d + \Delta(G) - 1$ if $\Delta(G)$ is even, or $\lambda_d(G(P_{3q+4})) \leq d + \Delta(G)$ if $\Delta(G)$ is odd. Trivially, $\lambda_d(G(P_{3q+4})) \geq \Delta(G(P_{3q+4})) + d - 1 = \Delta(G) + d - 1$ so we conclude that $\lambda_d(G(P_{3q+4})) = d + \Delta(G) - 1$ if $\Delta(G)$ is even. □

The case $\Delta(G) = 2$ in Conjecture 1.1 follows from setting $d = 2$ and $q = 0$ in Corollary 2.5, and the remaining cases follow from setting $d = 2$ and $q = 0$ in Corollary 2.6 since if $\Delta(G) \geq 3$, then $\max\{\lceil \Delta(G)/2 \rceil, d\} = \lceil \Delta(G)/2 \rceil$; in addition, $\lambda_2(G(P_4)) = \Delta(G) + 1$ when $\Delta(G)$ is even and different from 2.

In what follows, Theorem 2.7 and Corollary 2.8 are similar to Theorem 2.4 and Corollary 2.5, respectively. Since the proofs are also analogous, some of their details will be left to the reader for the sake of brevity.

Theorem 2.7 *Let G be an arbitrary graph and let* $d \geq 2$ *be an integer. If* $k = 5$ *or 6, then*

i. $\lambda_d(G(P_k)) \leq d + 2$ *if* $\Delta(G) \leq 2$, *and*
ii. $\lambda_d(G(P_k)) \leq d + \lceil \Delta(G)/2 \rceil + \max\{\lceil \Delta(G)/2 \rceil, d\}$ *otherwise.*

Proof Item i. can be verified in the same manner as in the proof of Theorem 2.4 i. Suppose on the other hand that $\Delta(G) \geq 3$ and let $h = \lceil \Delta(G)/2 \rceil$ and $m = \max\{\lceil \Delta(G)/2 \rceil, d\}$. When $k = 5$ or 6, arguments similar to the ones in the proof of Theorem 2.4 ii. for $k = 4$, using the label patterns in Table 2 or Table 3, respectively, in lieu of Table 1, can be used to construct L(d,1)-labelings of $G(P_k)$ using labels in 0, 1, …, $d + h + m$ so $\lambda_d(G(P_k)) \leq d + h + m$ (for each table, observe that all the second and all the last labels in each of the repeating label patterns in the second column are different). □

Corollary 2.8 *Let G be an arbitrary graph and let* $d \geq 2$ *be an integer. If* $p = 5$ *or* 6, *then, for any integer* $q \geq 0$,

Table 2 Labeling summary in the proof of Theorem 2.7 for $G(P_5)$ where $h = \lceil \Delta(G)/2 \rceil$ and $m = \max\{\lceil \Delta(G)/2 \rceil, d\}$

Subgraph of $G(P_5)$	Repeating label pattern for paths/cycles			
$G_1(P_5)$	0,	$d+1$,	1,	$d+m+1$
$G_2(P_5)$	0,	$d+2$,	1,	$d+m+2$
$G_3(P_5)$	0,	$d+3$,	1,	$d+m+3$
…	…	…	…	…
$G_{h-1}(P_5)$	0,	$d+h-1$,	1,	$d+h+m-1$
$G_h(P_5)$	0,	$d+h$,	1,	$d+h+m$

Table 3 Labeling summary in the proof of Theorem 2.7 for $G(P_6)$ where $h = \lceil \Delta(G)/2 \rceil$ and $m = \max\{\lceil \Delta(G)/2 \rceil, d\}$

Subgraph of $G(P_6)$	Repeating label pattern for paths/cycles				
$G_1(P_6)$	0,	$d+1$,	1,	$\boldsymbol{d+2}$,	$\boldsymbol{d+m+2}$
$G_2(P_6)$	0,	$d+2$,	1,	$d+1$,	$\boldsymbol{d+m+1}$
$G_3(P_6)$	0,	$d+3$,	1,	$d+1$,	$d+m+3$
$G_3(P_6)$	0,	$d+4$,	1,	$d+1$,	$d+m+4$
...	...	...	...	...	...
$G_{h-1}(P_6)$	0,	$d+h-1$,	1,	$d+1$,	$d+h+m-1$
$G_h(P_6)$	0,	$d+h$,	1,	$d+1$,	$d+h+m$

Note that the labels in bold do not follow their respective column pattern

i. $\lambda_d(G(P_{3q+p})) \leq d+2$ *if* $\Delta(G) \leq 2$, *and*
ii. $\lambda_d(G(P_{3q+p})) \leq d + \lceil \Delta(G)/2 \rceil + \max\{\lceil \Delta(G)/2 \rceil, d\}$ *otherwise.*

Proof Let us first assume that $k = 3q + 5$ for some nonnegative integer q. If $\Delta(G) \leq 2$, then $\lambda_d(G(P_{3q+5})) \leq d+2$ by an argument similar to the one used in the beginning of the proof of Theorem 2.4 i. Suppose on the other hand that $\Delta(G) \geq 3$ and consider the labeling of $G(P_5)$ constructed in the proof of Theorem 2.7 ii. where $h = \lceil \Delta(G)/2 \rceil$ and $m = \max\{\lceil \Delta(G)/2 \rceil, d\}$. For a fixed edge e of G, let $(0, x, y, z)$ be the label pattern which was used to label the first 4 vertices in P_5 that replaced e in $G(P_5)$; use this same label pattern followed by q repetitions of the label pattern $(0, d, z)$ to label the first $3q + 4$ vertices of P_{3q+5} that replaces e in $G(P_{3q+5})$ in order, from left to right along the path, and label the final vertex in this path with a 0. As in the proof of Theorem 2.4 ii., it can be shown that this labeling of $G(P_{3q+5})$ is an L(d,1)-labeling using labels in 0, 1, …, $d+h+m$ so $\lambda_d(G(P_{3q+5})) \leq d+h+m$. Similar arguments can be used if $k = 3q + 6$ for some nonnegative integer q to obtain $\lambda_d(G(P_{3q+6})) \leq d+h+m$ (in the argument above, replace each occurrence of 4, 5, $(0, x, y, z)$, Table 2 with 5, 6, $(0, w, x, y, z)$, Table 3, respectively). □

We close by noticing that Conjecture 1.2 is true for $G(P_k)$ and any integer $k \geq 4$ since, from Corollary 2.5 and Corollary 2.8,

i. $\lambda_d(G(P_k)) \leq d+2 \leq \Delta(G)+2d-1$ if $\Delta(G) \leq 2$, and
ii. $\lambda_d(G(P_k)) \leq d + \lceil \Delta(G)/2 \rceil + \max\{\lceil \Delta(G)/2 \rceil, d\} \leq \Delta(G)+2d-1$ otherwise.

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References

Adams SS, Booth P, Jaffe H, Zinnen SL (2012) Exact λ-numbers of generalized Petersen graphs of certain higher orders and on Möbius strips. Discret Appl Math 160:436–447

Calamoneri T (2011) The L(h, k)-labelling problem: a survey and annotated bibliography. Comput J 54:1344–1371

Cerioli MR, Posner DFD (2012) On L(2,1)-coloring split, chordal bipartite and weakly chordal graphs. Discret Appl Math 160:2655–2661

Charpentier C, Montassier M, Raspauld A (2012) L(p, q)-labeling of sparse graphs. J Comb Optim. doi:10.1007/s10878-012-9507-6

Chia ML, Kuo D, Yan JH, Yang SR (2012) (p, q)-total labeling of complete graphs. J Comb Optim. doi:10.1007/s10878-012-9471-1

Georges JP, Mauro DW, Whittlesey MA (1994) Relating path coverings to vertex labelings with a condition at distance two. Discret Math 135:103–111

Georges JP, Mauro DW (1995) Generalized vertex labeling with a condition at distance two. Congr Numer 109:141–159

Gonçalves D (2008) On the L(p,1)-labelling of graphs. Discret Math 308:1405–1414

Griggs JR, Yeh RK (1992) Labeling graphs with a condition at distance two. SIAM J Discret Math 5:585–595

Havet F, Reed B, Sereni JS- (2012) Griggs and Yeh's conjecture and L(p,1)-labelings. SIAM J Discret Math 26:145–168

Havet F, Yu ML (2008) (p, 1)-Total labeling of graphs. Discret Math 308:498–513

Lin W, Wu J (2012) Distance two edge labelings of lattices. J Comb Optim. doi:10.1007/s10878-012-9508-5

Lü D (2012) L(2,1)-labelings of the edge-path-replacement of a graph. J Comb Optim. doi:10.1007/s10878-012-9470-2

Lü D, Lin N (2012) L(d,1)-labelings of the edge-path-replacement of a graph. J Comb Optim. doi:10.1007/s10878-012-9487-6

Panda BS, Goel P (2012) L(2, 1)-labeling of dually chordal graphs and strongly orderable graphs. Info Proc Lett 112:552–556

Petersen J (1891) Die theorie der regularen graphen. Acta Math 15:193–220

Wang F, Lin W (2012) Group path covering and L(j, k)-labelings of diameter two graphs. Info Proc Lett 112:124–128

Wu Q, Shiu WC, Sun PK (2012) Circular L(j, k)-labeling number of direct product of path and cycle. J Comb Optim. doi:10.1007/s10878-012-9520-9

Yeh RK (2006) A survey on labeling graphs with a condition at distance two. Discret Math 306:1217–1231

Zhai M, Lu C, Shu J (2012) A note on L(2, 1)-labelling of trees. Acta Mathematicae Applicatae Sinica 28:395–400